W² Education

This document reflects the typical format of an official HSC paper, complete with solutions and supplementary comments. While originally created to support our own students, it is now made available to a wider audience, offering valuable insights for anyone seeking to improve their approach.

2026 COMPREHENSIVE EXAM REVIEW – DETAILED SOLUTIONS & COMMENTS

Mathematics Extension 1

General Comments

- Original examination items are NOT included
- Comments and postmortem insights are shown in blue
- This document features suggested solutions which do NOT serve as official solutions for this paper
- All examination content is © NESA and is used for educational purposes. We do NOT own any exam content and aim solely to provide helpful insights
- For any enquiries or concerns, please refer to the admin contact details at the end of this document

Total marks: 70

Section I – 10 marks (pages 2–3)

- Answers for Questions 1–10
- Allow about 15 minutes for this section

Section II – 60 marks (pages 4–23)

- Solutions for Questions 11–14
- Allow about 1 hour and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

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1 A.

As $\sin\alpha\cos\alpha = \frac{1}{2}\sin 2\alpha$ and $\sin 2\alpha \le 1$, the maximum value is $\frac{1}{2}$.

2 D.

The derivative approaches infinity and is undefined at $x = \frac{3}{2}$.

3 C.

We need $f(x) > 0$ for $x \in (-2, 0) \cup (5, \infty)$ and $f(x) = 0$ at $x = -2, 0, 5$.

4 D.

We need $\frac{dy}{dx} > 0$, which occurs at points C and D. At C, the curve has positive concavity.

5 C.

From $x = t^2 - 3$, we need $x \ge -3$.

6 B.

As $g(-6) = 2$, $f(2) = -6$ so $g'(-6) = \frac{1}{f'(2)} = \frac{1}{3(2^2) - 7} = \frac{1}{5}$.

Unrestricted, f is NOT one-to-one, and thus f does NOT admit an inverse function on its whole domain. The question implies a restricted domain such as $x \ge \sqrt{\frac{7}{3}}$, which contains $x = 2$.

A question on the derivative of an inverse function at a point was predicted in the multiple-choice section of our **2026 HSC Masterclass** HSC-level practice paper.

7 B.

We can confirm that $S(1)$ is true and $S(2)$ is false. That is, $S(1)$ being true does NOT imply that $S(2)$ is true.

8 D.

For the path to be a circle, $\left|-\sqrt{a}\right| = \left|-\frac{1}{b}\right|$. Squaring gives $a = \frac{1}{b^2}$.

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9 A.

The basket holds $n+1$ different kinds of sock, so at most $n+1$ socks can be selected without a matching pair.

As 6 socks do not guarantee a pair, $n+1 \geq 6$. But 7 socks do, so $n+1 \leq 6$.

Hence $n+1=6$, and so $n=5$.

Alternatively, we can affirm A. by considering the pigeonhole principle on the scenario $0, (1,1), (2,2), (3,3), (4,4), (5,5)$ $(n=5)$.

10 A.

Out of ${}^{46}C_{10}$ possible draws, the number that win a prize is ${}^{40}C_4$, which is the number of ways the four balls which are NOT within the player's selections can be chosen.

Alternative method

With $n=46$ balls, $k=10$ drawn and $r=6$ chosen, A. is $\frac{{}^{n-r}C_{k-r}}{{}^{n}C_{k}}$. If all balls are drawn so that $k=n$, the player certainly wins, and A. becomes $\frac{{}^{n-r}C_{n-r}}{{}^{n}C_{n}}=1$, while B., C. and D. do NOT equal 1 in general.

Hence, we eliminate B., C. and D..

Alternative method (credit to our student ZL)

Consider the player's six numbers one at a time. The first is among the ten balls drawn with probability $\frac{10}{46}$. Given this, the second is among the other nine balls drawn with probability $\frac{9}{45}$, and so on.

So the probability is $\frac{10}{46}\times\frac{9}{45}\times\frac{8}{44}\times\frac{7}{43}\times\frac{6}{42}\times\frac{5}{41}=\frac{10}{446\,039}=\frac{{}^{40}C_4}{{}^{46}C_{10}}$.

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Section II

60 marks
Attempt Questions 11–14
Allow about 1 hour and 45 minutes for this section

Answer the questions in the Section II Writing Booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer Question 11 on pages 2–5 of the Writing Booklet.

(a) Write $P(x) \equiv (3x-1)(3x+1)\,Q_1(x) + 3x + 7$, where $Q_1(x)$ is the quotient when $P(x)$ is divided by $9x^2 - 1$. **2**

Divide $P(x)$ by $3x-1$ by writing $P(x) \equiv (3x-1)\,Q_2(x) + R$, where $Q_2(x)$ is the quotient and R is the constant remainder, as $3x-1$ is linear.

Put $x = \frac{1}{3}$, so $R = P\left(\frac{1}{3}\right) = 3\left(\frac{1}{3}\right) + 7$.

Hence the remainder is 8.

Alternative method

As $3x + 7 = (3x-1) + 8$, $P(x) \equiv \big(3x-1\big)\big((3x+1)\,Q_1(x) + 1\big) + 8$.

Hence, as 8 is a constant and $3x-1$ is linear, the remainder is 8.

Though slightly imprecise, it probably would have been okay to use the remainder theorem with the *non-monic* linear factor $(3x-1)$.

Question 11 continues on page 5

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(b) Consider $\dfrac{x+6}{x-3} = \dfrac{(x-3)+9}{x-3} > 4$ **3**

$$1 + \frac{9}{x-3} > 4$$

$$\frac{9}{x-3} > 3 > 0, \text{ so } x-3>0, \text{ i.e., } x>3.$$

As $x-3>0$, multiplying by $(x-3)$,

$$9 > 3(x-3)$$

$$x < 6.$$

Hence $x \in (3, 6)$.

Alternative method

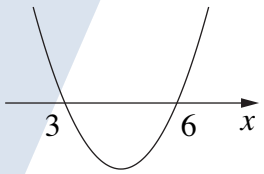
As $(x-3)^2 > 0$ for $x \neq 3$, multiply both sides by $(x-3)^2$.

So

$$(x+6)(x-3) > 4(x-3)^2$$

$$(x-3)(4x-12-x-6) < 0$$

$$3(x-3)(x-6) < 0$$



Hence $x \in (3, 6)$.

(c) For $y(x)$, both $-1 \leq \dfrac{x}{3} \leq 1$ and $-\dfrac{\pi}{2} \leq \dfrac{y}{2} \leq \dfrac{\pi}{2}$. **2**

So $-3 \leq x \leq 3$ and $-\pi \leq y \leq \pi$.

Hence the domain is $[-3, 3]$ and the range is $[-\pi, \pi]$.

Question 11 continues on page 6

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(d) For $\begin{pmatrix} m \\ -8 \end{pmatrix} \| \begin{pmatrix} m-2 \\ 1 \end{pmatrix}$, **2**

$$\frac{m-2}{m} = 1 - \frac{2}{m} = \frac{1}{-8}, \; m \neq 0$$

$$m = \frac{2}{1 + \frac{1}{8}}$$

$$= \frac{16}{9}.$$

Hence $m = \frac{16}{9}$.

Alternative method

As the vectors are parallel, $\begin{pmatrix} m \\ -8 \end{pmatrix} = \lambda \begin{pmatrix} m-2 \\ 1 \end{pmatrix}$, for some real λ.

So $\lambda = -8$ and $m = -8(m-2)$

$$= -8m + 16$$

$$9m = 16.$$

Hence $m = \frac{16}{9}$.

Question 11 continues on page 7

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(e) Consider $2\cos^3 x - 2\cos^2 x - \cos x + 1 = 2\cos^2 x(\cos x - 1) - (\cos x - 1)$ **3**

$= (\cos x - 1)(2\cos^2 x - 1).$

So $\cos x = 1$ or $\cos 2x = 0.$

Thus $x = 0$ or $2x = \pm\frac{\pi}{2}$, for $-\frac{\pi}{2} \le x \le \frac{\pi}{2}.$

Hence $x = -\frac{\pi}{4}, 0, \frac{\pi}{4}.$

Question 11 continues on page 8

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(f) With $u = x^2$, $du = 2x\,dx$. **3**

At $x = 0$, $u = 0$, and at $x = \dfrac{1}{\sqrt{2}}$, $u = \dfrac{1}{2}$.

So

$$\int_0^{\frac{1}{\sqrt{2}}} \frac{x}{\sqrt{1-x^4}}\,dx = \frac{1}{2}\int_0^{\frac{1}{\sqrt{2}}} \frac{2x\,dx}{\sqrt{1-\left(x^2\right)^2}}$$

$$= \frac{1}{2}\int_0^{\frac{1}{2}} \frac{du}{\sqrt{1-u^2}}$$

$$= \frac{1}{2}\sin^{-1}u\Big|_0^{\frac{1}{2}}$$

$$= \frac{1}{2}\left(\frac{\pi}{6} - 0\right)$$

$$= \frac{\pi}{12}.$$

This exact definite integral was set as Question 11 (b) of our **2026 Trial Premiere**.

End of Question 11

Education

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Question 12 (15 marks) Answer Question 12 on pages 7–10 of the Writing Booklet.

(a) Let X be the number of the 20 people with blood type B+, so $X \sim \text{Bin}(20, 0.12)$. **2**

Then

$$\begin{aligned}\Pr(X \geq 2) &= 1 - \Pr(X = 0) - \Pr(X = 1) \\ &= 1 - (0.88)^{20} - 20(0.12)(0.88)^{19} \\ &= 0.7109\ldots \\ &\approx 0.711 \text{ (3 d.p.).}\end{aligned}$$

(b) For $n = 1$, $n\left(n^2 + 2\right) = 1(1 + 2) = 3$, which is divisible by 3. **3**

The statement is true for $n = 1$.

Assume true for $n = k$, where k is a positive integer, so

$$k\left(k^2 + 2\right) = 3P, \text{ where } P \in \mathbb{Z}$$

to prove also true for $n = k + 1$, i.e.,

$$(k + 1)\left(k^2 + 2k + 3\right) = 3Q, \text{ where } Q \in \mathbb{Z}.$$

Consider $(k + 1)\left(k^2 + 2k + 3\right) = \left(k^3 + 2k\right) + 3k^2 + 3k + 3$

$$\begin{aligned}&= 3P + 3\left(k^2 + k + 1\right), \text{ using the assumption} \\ &= 3\left(P + k^2 + k + 1\right) \\ &= 3Q, \text{ where } Q = P + k^2 + k + 1 \in \mathbb{Z}, \text{ as } P, k \in \mathbb{Z}.\end{aligned}$$

Hence the statement is true by mathematical induction for $n \geq 1$.

Question 12 continues on page 10

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(c) Consider **2**

$$\begin{aligned}
\left|\underset{\sim}{a}-2\underset{\sim}{b}\right|^2 &= \left(\underset{\sim}{a}-2\underset{\sim}{b}\right)\cdot\left(\underset{\sim}{a}-2\underset{\sim}{b}\right)\\
&= \underset{\sim}{a}\cdot\underset{\sim}{a}-\underset{\sim}{a}\cdot\left(2\underset{\sim}{b}\right)-\left(2\underset{\sim}{b}\right)\cdot\underset{\sim}{a}+4\underset{\sim}{b}\cdot\underset{\sim}{b}\\
&= \left|\underset{\sim}{a}\right|^2-4\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right)+4\left|\underset{\sim}{b}\right|^2\\
&= 4-4(3)+4(9)\\
&= 2^2(7).
\end{aligned}$$

Hence $\left|\underset{\sim}{a}-2\underset{\sim}{b}\right| = 2\sqrt{7}$, as $\left|\underset{\sim}{a}-2\underset{\sim}{b}\right| \geq 0$.

A near-identical question appeared in our **2024 HSC Premiere** (Question 11 (e)), differing only in that both vectors were unit vectors at $60°$ and the expansion of $\left|\underset{\sim}{u}-\underset{\sim}{v}\right|^2$ was given.

(d) With $\alpha = \tan^{-1}(5x)$ and $\beta = \tan^{-1}(3x)$, solve $\alpha-\beta = \tan^{-1}\left(\frac{1}{4}\right)$, where $\tan\alpha = 5x$ and $\tan\beta = 3x$. **2**

Then

$$\begin{aligned}
\tan(\alpha-\beta) &= \frac{5x-3x}{1+(5x)(3x)} = \frac{1}{4}\\
\frac{2x}{1+15x^2} &= \frac{1}{4}\\
15x^2-8x+1 &= 0\\
(1-3x)(1-5x) &= 0.
\end{aligned}$$

Hence $x = \frac{1}{3}$ or $x = \frac{1}{5}$.

A very common question type drilled countless times in exam training class tests. In fact this HSC question is one of the simplest types because no extraneous solutions were introduced by a naïve method.

Question 12 continues on page 11

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(e) As $\cos\frac{\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\begin{pmatrix} m \\ 2 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 3 \end{pmatrix}}{\sqrt{m^2+4}\sqrt{10}}$, **3**

$$10\left(m^2+4\right) = 2(m+6)^2,\ m \geq -6$$
$$4m^2 - 12m - 16 = 0$$
$$4(m-4)(m+1) = 0.$$

Hence $m = 4$ or $m = -1$.

A near-identical question was predicted in our **2026 HSC Masterclass** HSC-level practice paper (Question 11 (b)), but a more conceptually layered question since one root of the resulting quadratic had to be rejected.

(f) (i) As E is the midpoint of AB, **1**

$$\begin{aligned}\overrightarrow{OE} &= \overrightarrow{OA} + \frac{1}{2}\left(\overrightarrow{OB} - \overrightarrow{OA}\right) \\ &= \frac{1}{2}\left(\overrightarrow{OA} + \overrightarrow{OB}\right) \\ &= \frac{1}{2}\begin{pmatrix} 6\cos\theta \\ 6\sin\theta \end{pmatrix} \\ &= 3\begin{pmatrix} \cos\theta \\ \sin\theta \end{pmatrix}.\end{aligned}$$

Question 12 continues on page 12

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(f) (ii) As $\left|\overrightarrow{OC}\right| = 5$ (radius) and $\overrightarrow{OC} \cdot \overrightarrow{OE} > 0$, $\overrightarrow{OC} = 5\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$, **2**

so $\overrightarrow{OD} = -\overrightarrow{OC} = -5\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$.

As $\overrightarrow{AE} = \begin{pmatrix}3\cos\theta \\ 3\sin\theta\end{pmatrix} - \begin{pmatrix}3\cos\theta - 4\sin\theta \\ 3\sin\theta + 4\cos\theta\end{pmatrix} = 4\begin{pmatrix}\sin\theta \\ -\cos\theta\end{pmatrix}$, $\left|\overrightarrow{AE}\right| = 4$, as $\sqrt{\sin^2\theta + \cos^2\theta} = 1$.

Now

$$\overrightarrow{CE} = 3\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} - 5\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} = -2\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}$$

$$\overrightarrow{DE} = 3\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} + 5\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix} = 8\begin{pmatrix}\cos\theta \\ \sin\theta\end{pmatrix}.$$

Hence $\left|\overrightarrow{CE}\right|\left|\overrightarrow{DE}\right| = 2 \times 8 = 16 = 4^2 = \left|\overrightarrow{AE}\right|^2$.

Alternative method

By (i), $\left|\overrightarrow{OE}\right| = 3$. As E lies on the diameter CD and $3 < 5$, by the diagram $\left|\overrightarrow{CE}\right| = 5 - 3 = 2$ and $\left|\overrightarrow{DE}\right| = 5 + 3 = 8$. As $CD \perp AB$, triangle OEA is right-angled at E, so by Pythagoras' theorem,

$$\left|\overrightarrow{AE}\right|^2 = \left|\overrightarrow{OA}\right|^2 - \left|\overrightarrow{OE}\right|^2$$
$$= 5^2 - 3^2 = 16 = 2 \times 8 = \left|\overrightarrow{CE}\right|\left|\overrightarrow{DE}\right|.$$

End of Question 12

Education

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Question 13 (15 marks) Answer Question 13 on pages 12–15 of the Writing Booklet.

(a) Let X be the number of correct guesses, so $X \sim \text{Bin}\left(n, \frac{1}{4}\right)$. Then $\hat{p} = \frac{X}{n}$ follows approximately $N\left(\frac{1}{4}, \left(\sqrt{\frac{3}{16n}}\right)^2\right)$. **3**

For $\Pr(\hat{p} \geq 0.2) \approx \Pr\left(Z \geq \frac{0.2 - 0.25}{\sqrt{3}} \times 4\sqrt{n}\right) \geq 0.9901$,

$$\Pr\left(Z \leq \frac{4(0.05)}{\sqrt{3}}\sqrt{n}\right) \geq 0.9901.$$

By the table, $0.2\sqrt{\frac{n}{3}} \geq 2.33 > 0$, so $n \geq 3\left(\frac{2.33}{0.2}\right)^2 = 407.1675$.

Hence the smallest such n is 408.

The wording of the question is ambiguous. The value of n that yields the closest numerical probability to 99.01% is $n = 407$. Yet the word 'smallest' suggests that the objective is to find n which yields *at least* a 99.01% chance of passing.

(b) Consider $(m+n)(m+n-1) = m^2 + 2mn + n^2 - m - n$ **2**

$$= m(m-1) + n(n-1) + 2mn.$$

So $\frac{(m+n)(m+n-1)}{2} = \frac{m(m-1)}{2} + \frac{n(n-1)}{2} + mn.$

Hence $\binom{m+n}{2} = \binom{m}{2} + \binom{n}{2} + \binom{m}{1}\binom{n}{1}$, as $m, n \in \mathbb{Z}^+$.

Question 13 continues on page 14

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(b) **Alternative method**

Consider choosing 2 kids from a group of $m + n$ kids, consisting of m boys and n girls. This can be done in $\binom{m+n}{2}$ ways.

Consider the possible gender(s):

For 2 boys, there are $\binom{m}{2}$ ways.

For 1 boy and 1 girl, there are $\binom{m}{1}\binom{n}{1}$ ways.

For 2 girls, there are $\binom{n}{2}$ ways.

Hence $\binom{m+n}{2} = \binom{m}{2} + \binom{n}{2} + \binom{m}{1}\binom{n}{1}$.

Question 13 continues on page 15

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(c) As $y = \frac{5}{9}x^2$, $x^2 = \frac{9y}{5}$. For water of height h, **4**

$$V = \pi \int_0^h \frac{9y}{5}\,dy$$
$$= \frac{9\pi}{10} y^2 \Big|_0^h$$
$$= \frac{9\pi}{10}\left(h^2 - 0\right) = \frac{9\pi h^2}{10}.$$

At $h = 10$, the full capacity is 90π cm^3.

Now $\frac{dV}{dh} = \frac{9\pi h}{5}$.

Consider $\frac{dV}{dh} \times \frac{dh}{dt} = \frac{dV}{dt}$, by the chain rule
$= 15$ (given).

So $\frac{dh}{dt} = 15 \times \frac{5}{9\pi h} = \frac{25}{3\pi h}$.

At half capacity, $\frac{9\pi h^2}{10} = \frac{90\pi}{2}$, so $h^2 = 50$ and $h = \sqrt{50}$, as $h > 0$.

So at half capacity, $\frac{dh}{dt} = \frac{25}{15\pi\sqrt{2}} = \frac{5}{3\pi\sqrt{2}}$.

Hence the height rises at $\frac{5}{3\pi\sqrt{2}}$ cm s^{-1}.

A similar question was predicted in our **2026 HSC Masterclass** HSC-level practice paper in Question 12 (d), involving a solid of revolution container filled at a constant rate, asking for the rate at which the depth of the water changes.

Question 13 continues on page 16

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(d) (i) The particle hits P when $20t\cos\theta = \ell\cos 45°$, as $\ell = OP$, so $t = \dfrac{\ell}{20\sqrt{2}\cos\theta}$. **3**

But at P, $\underset{\sim}{r}(t)\cdot\underset{\sim}{i} = \underset{\sim}{r}(t)\cdot\underset{\sim}{j}$, as the plane is the line $y = x \geq 0$.

So

$$20t\cos\theta = 20t\sin\theta - 5t^2$$

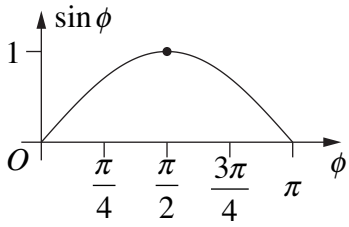
$$t = 4(\sin\theta - \cos\theta) = \frac{\ell}{20\sqrt{2}\cos\theta}, \text{ as } t > 0.$$

Hence $\ell = 80\sqrt{2}\cos\theta(\sin\theta - \cos\theta)$.

(ii) By (i), **3**

$$\begin{aligned}
\ell &= 80\sqrt{2}\cos\theta(\sin\theta - \cos\theta) \\
&= 40\sqrt{2}\left(2\sin\theta\cos\theta - \left(2\cos^2\theta - 1\right) - 1\right) \\
&= 40\sqrt{2}(\sin 2\theta - \cos 2\theta - 1) \\
&= 80\left(\frac{1}{\sqrt{2}}\sin 2\theta - \frac{1}{\sqrt{2}}\cos 2\theta - \frac{1}{\sqrt{2}}\right) \\
&= 80\left(\sin 2\theta\cos\frac{\pi}{4} - \cos 2\theta\sin\frac{\pi}{4} - \frac{1}{\sqrt{2}}\right) \\
&= 80\left(\sin\left(2\theta - \frac{\pi}{4}\right) - \frac{1}{\sqrt{2}}\right).
\end{aligned}$$

So ℓ is a maximum when $\sin\left(2\theta - \dfrac{\pi}{4}\right) = 1 = \sin\dfrac{\pi}{2}$.



As $\dfrac{\pi}{4} < \theta < \dfrac{\pi}{2}$, $\dfrac{\pi}{4} < 2\theta - \dfrac{\pi}{4} < \dfrac{3\pi}{4}$,

so $2\theta - \dfrac{\pi}{4} = \dfrac{\pi}{2}$ and $2\theta = \dfrac{3\pi}{4}$.

Hence $\theta = \dfrac{3\pi}{8}$.

Question 13 continues on page 17

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(d) (ii) **Alternative method**

From $\ell = 40\sqrt{2}(\sin 2\theta - \cos 2\theta - 1)$,

$$\frac{d\ell}{d\theta} = 80\sqrt{2}(\cos 2\theta + \sin 2\theta)$$
$$= 0, \text{ when } \tan 2\theta = -1.$$

As $\frac{\pi}{2} < 2\theta < \pi$, $2\theta = \frac{3\pi}{4}$, so $\theta = \frac{3\pi}{8}$, and here $\frac{d^2\ell}{d\theta^2} = 160\sqrt{2}(\cos 2\theta - \sin 2\theta) = -320 < 0$, so this is indeed a maximum.

This is a very standard question, and has been covered extensively several times during our classes on *Projectile Motion* in Mathematics Extension 1.

The heuristic is recognising that the range up the incline is greatest when the angle of projection bisects the angle between the incline and the vertical. In this case, $\theta = \frac{1}{2}\left(\frac{\pi}{4} + \frac{\pi}{2}\right) = \frac{3\pi}{8}$.

More general versions of this question have appeared in past HSC exams too.

End of Question 13

Education

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Question 14 (15 marks) Answer Question 14 on pages 17–20 of the Writing Booklet.

(a) Firstly, as f is increasing, it is one-to-one, so g exists, and $f\big(g(x)\big) \equiv x$. **2**

Differentiating both sides with respect to x, $f'\big(g(x)\big)g'(x) \equiv 1$, so $g'(x) = \dfrac{1}{f'\big(g(x)\big)}$.

Once more,

$$g''(x) = -\frac{f''\big(g(x)\big)g'(x)}{\big(f'\big(g(x)\big)\big)^2}.$$

But $f''\big(g(x)\big) > 0$, as f is concave up, and $g'(x) > 0$, since $f'\big(g(x)\big) > 0$, as f is increasing.

So $g''(x) < 0$, as $\big(f'\big(g(x)\big)\big)^2 > 0$.

Hence $g(x)$ is concave down.

(b) By separation of variables, **3**

$$\int \frac{dy}{y} = \int \sec^2 x\, dx, \text{ as } y \neq 0$$

$$\ln|y| = \tan x + c$$

$$y = Ae^{\tan x}, \text{ where } A = \pm e^c \in \mathbb{R}.$$

For $y\left(\dfrac{\pi}{4}\right) = -e$, $Ae = -e$, so $A = -1$.

Hence $y = -e^{\tan x}$, with $f(x) = -e^{\tan x}$.

Question 14 continues on page 19

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(c) Consider $3\cos x + (m-3)\sin x = 5$. **3**

By auxiliary angle, $R > 0$ and $\alpha \in \mathbb{R}$ can be found such that

$$3\cos x + (m-3)\sin x \equiv R\cos(x-\alpha).$$

Compare the LHS with RHS $= (R\cos\alpha)\cos x + (R\sin\alpha)\sin x$, and let $R\cos\alpha = 3$ and $R\sin\alpha = m-3$.

So by squaring then adding,

$$R^2\left(\cos^2\alpha + \sin^2\alpha\right) = R^2 = 9 + (m-3)^2$$
$$R = \sqrt{9+(m-3)^2}, \text{ as } R > 0.$$

The range of $R\cos(x-\alpha)$ is $[-R, R]$, as $|\cos(x-\alpha)| \le 1$, so if $R \ge 5$, then $R\cos(x-\alpha) = 5$ will always have solutions.

Thus, for no solutions, $R < 5$.

So

$$0 < \sqrt{9+(m-3)^2} < 5$$
$$9 + (m-3)^2 < 25$$
$$(m-3)^2 < 16 = 4^2.$$

So $|m-3| < 4$, that is, $-4 < m-3 < 4$.

Hence $m \in (-1, 7)$.

Question 14 continues on page 20

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(c) **Alternative method**

Let $t = \tan\dfrac{x}{2}$, so $\cos x = \dfrac{1-t^2}{1+t^2}$ and $\sin x = \dfrac{2t}{1+t^2}$, for $x \neq \pi + 2k\pi$, where $k \in \mathbb{Z}$.

Then

$$3\left(1-t^2\right) + 2(m-3)t = 5\left(1+t^2\right)$$
$$4t^2 - (m-3)t + 1 = 0.$$

For no solutions, $\Delta = (m-3)^2 - 16 < 0$.

Hence $-1 < m < 7$.

It is easy, but unnecessary, to check that $x = \pi$ is not a solution.

Alternative method

Let $\underset{\sim}{u} = \begin{pmatrix} 3 \\ m-3 \end{pmatrix}$ and $\underset{\sim}{v} = \begin{pmatrix} \cos x \\ \sin x \end{pmatrix}$, so the equation becomes $\underset{\sim}{u} \cdot \underset{\sim}{v} = 5$, where $\left|\underset{\sim}{v}\right| = 1$.

As $\underset{\sim}{u} \cdot \underset{\sim}{v} = \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right|\cos\theta_{\underset{\sim}{u},\,\underset{\sim}{v}}$, for $x \in \mathbb{R}$, $\left|\underset{\sim}{u} \cdot \underset{\sim}{v}\right| \leq \left|\underset{\sim}{u}\right|\left|\underset{\sim}{v}\right| = \left|\underset{\sim}{u}\right|$. Thus, for no solutions, we set $\left|\underset{\sim}{u}\right| < 5$. So $9 + (m-3)^2 < 25$.

Hence $-1 < m < 7$.

This is essentially the auxiliary angle method expressed using the Cauchy–Schwarz inequality, which is in the new Mathematics Extension 2 syllabus for 2027.

Question 14 continues on page 21

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(d) Note that $P(t) = 5000$ is a constant solution. But $P(0) = 7000 > 5000$, so $P(t) > 5000$ for $t \geq 0$. **4**

Separating variables, $\int \frac{5000dP}{P(5000 - P)} = \int dt.$

By the given result,

$$\int \left(\frac{1}{P} + \frac{1}{5000 - P}\right) dP = \int dt$$

$$\ln\left(\frac{P}{P - 5000}\right) = t + c, \text{ as } P > 5000.$$

When $t = 0$, $P = 7000$, so $c = \ln\left(\frac{7000}{2000}\right) = \ln\left(\frac{7}{2}\right)$.

So for $P = 5100$,

$$t = \ln\left(\frac{5100}{100}\right) - \ln\left(\frac{7}{2}\right)$$

$$= \ln\left(\frac{102}{7}\right).$$

Hence it takes $\ln\left(\frac{102}{7}\right)$ years.

Justification for why $P > 5000$ should be given. Alternatively, the absolute values can be absorbed by another constant, say A, using the method in **Question 14 (b)**.

Question 14 continues on page 22

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(e) Consider $y' = \tan^{-1}x + \dfrac{x}{1+x^2}$. **3**

We show that $y = x\tan^{-1}x$ has only one stationary point at $x = 0$.

At $x = 0$, $y' = \tan^{-1}0 + \dfrac{0}{1+0^2} = 0 + 0 = 0$, so at $x = 0$ there is a stationary point.

For $x > 0$, $y' = \tan^{-1}x + \dfrac{x}{1+x^2} > 0 + 0$, as $\tan^{-1}x > 0$ for $x > 0$, and $\dfrac{x}{1+x^2} > 0$, since $1 + x^2 > 0$.

So for $x > 0$, $y' > 0$, and thus $y' \neq 0$ for any $x > 0$.

Now $y(-x) = -x\tan^{-1}(-x)$

$= \left(-x\right)\left(-\tan^{-1}x\right)$, as $\tan^{-1}(\cdot)$ is odd

$= x\tan^{-1}x = y(x)$,

so $y = x\tan^{-1}x$ is even.

As $y = x\tan^{-1}x$ is even, $y' < 0$ for $x < 0$, so $y' \neq 0$ for any $x < 0$.

Thus $y' = 0$ at $x = 0$ ONLY.

Hence $y = x\tan^{-1}x$ has only one stationary point, namely at $x = 0$.

That the derivatives hold for all real x implies the functions are continuous.

Question 14 continues on page 23

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(e) **Alternative method**

As $y'' = \dfrac{1}{1+x^2} + \dfrac{\left(1+x^2\right) - 2x^2}{\left(1+x^2\right)^2} = \dfrac{2}{\left(1+x^2\right)^2} > 0$ for all $x \in \mathbb{R}$, y' is strictly increasing.

Hence, as $y'(0) = 0$, $y' = 0$ has exactly one solution, $x = 0$.

The alternative method uses the same idea as Question 13 (a) of our **2025 HSC Premiere**, where the second derivative was used to determine the sign of the first derivative. Though our question was more difficult, because it also layered in the concept of inverse functions as well as conditions for invertibility.

End of solutions

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