W² Education

This document reflects the typical format of an official HSC paper, complete with solutions and supplementary comments. While originally created to support our own students, it is now made available to a wider audience, offering valuable insights for anyone seeking to improve their approach.

2026 COMPREHENSIVE EXAM REVIEW – DETAILED SOLUTIONS & COMMENTS

Mathematics Extension 2

General Comments

- Original examination items are NOT included
- Comments and postmortem insights are shown in blue
- This document features suggested solutions which do NOT serve as official solutions for this paper
- All examination content is © NESA and is used for educational purposes. We do NOT own any exam content and aim solely to provide helpful insights
- For any enquiries or concerns, please refer to the admin contact details at the end of this document

Total marks: 100

Section I – 10 marks (pages 2–3)

- Answers for Questions 1–10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 4–30)

- Solutions for Questions 11–16
- Allow about 2 hours and 45 minutes for this section

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10.

---

1 A.

By sum of roots, $3 + i + \alpha + \beta = 10$, so $\alpha + \beta = 7 - i$. By product of roots, $(3 + i)\alpha\beta = 40$, so $\alpha\beta = \dfrac{40}{3 + i} = 4(3 - i)$. Hence we affirm A..

2 B.

The length is $\sqrt{4 + 9 + 36} = 7$ units.

3 D.

The coefficient of x needs to be positive and the coefficient of y needs to be negative.

4 D.

Expressing $\text{Re}(z) \geq \text{Im}(z)$ as $y \leq x$, the region lies below the line. Also, the centre of the circle lies in the first quadrant.

5 B.

As $\dfrac{w}{z} = 3 + 4i$, $\theta = \arg(w) - \arg(z) = \arg\left(\dfrac{w}{z}\right) + 2k\pi = \tan^{-1}\left(\dfrac{4}{3}\right) + 2k\pi$, where $k \in \mathbb{Z}$.

Hence $\sin\theta = \sin\left(\tan^{-1}\left(\dfrac{4}{3}\right)\right) = \dfrac{4}{5}$.

6 C.

The negation includes both $\exists p > 0$ and $q^2 \neq p$.

7 C.

By partial fractions, $\dfrac{7}{(x+1)(x-6)^2} \equiv \dfrac{1}{7(x+1)} - \dfrac{1}{7(x-6)} + \dfrac{1}{(x-6)^2}$, with all three terms non-zero.

8 B.

The triangle has $2|z|$ and $|z|$ as its base and height, as $-iz$ is a $90°$ rotation of z.

Hence the area is $\dfrac{1}{2} \times 2|z| \times |z| = |z|^2$.

Prepared by W² Education

9 A.

The lines are parallel because $\begin{pmatrix} -2 \\ 2 \\ -4 \end{pmatrix} = -2\begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

Also, the point $(3, 2, -2)$ is common to both lines with $(\lambda, \mu) = (1, 0)$.

This point can be considered to lie in the intersection of the set of points on ℓ_1 and the set of points on ℓ_2.

The lines ℓ_1 and ℓ_2 are in fact coincident. Though it is unusual to describe coincident lines as 'intersecting'. Were the answer NOT A., the intended answer would be B..

10 C.

For $f(x) = x^5 + x^4 = x^4(x+1)$, x divides $f''(x)$, so $f''(0) = 0$.

But its graph has a turning point at $x = 0$.

W2 Education

Section II

90 marks
Attempt Questions 11–16
Allow about 2 hours and 45 minutes for this section

Answer the questions in the Section II Writing Booklet. Extra writing booklets are available.

For questions in Section II, your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (15 marks) Answer Question 11 on pages 3–6 of the Writing Booklet.

(a) For integer(s) a, if a is odd then a^3 is odd. **1**

As written, the provided sentence is NOT a statement. The variable a should be quantified, whether universally ($\forall$) or existentially ($\exists$).

(b) (i) $3z + w = (9 + 6i) + (-1 - 5i) = 8 + i.$ **1**

(ii) $2\bar{z}w = 2(3 - 2i)(-1 - 5i)$ **1**

$= 2\left(-3 - 13i + 10i^2\right)$

$= 2(-13 - 13i)$

$= -26 - 26i.$

Question 11 continues on page 5

W2 Education

(c) Let $\underset{\sim}{a} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ and $\underset{\sim}{b} = \begin{pmatrix} 1 \\ 0 \\ 3 \end{pmatrix}$, so that θ is also the angle between $\underset{\sim}{a}$ and $\underset{\sim}{b}$, **2**

as $\underset{\sim}{b} \parallel \overrightarrow{OP_2}$ and $\underset{\sim}{b} \cdot \overrightarrow{OP_2} > 0$.

Then

$$\cos\theta = \frac{\underset{\sim}{a} \cdot \underset{\sim}{b}}{|\underset{\sim}{a}||\underset{\sim}{b}|}$$
$$= \frac{1}{\sqrt{10}}.$$

Hence $\tan^2\theta = \sec^2\theta - 1 = \dfrac{1}{\cos^2\theta} - 1 = 9$, as $\dfrac{1}{\cos\theta} = \sqrt{10}$.

(d) Consider $(6 + 5i)^2 = 36 - 25 + 2(6)(5i) = 11 + 60i$. **2**

Hence the square roots of $11 + 60i$ are $\pm(6 + 5i)$, with $(x, y) = (6, 5)$ or $(-6, -5)$.

(e) Write $z = r(\cos\theta + i\sin\theta)$, so $\bar{z} = r(\cos\theta - i\sin\theta)$, as $r,\ \theta \in \mathbb{R}$. **2**

So $z\bar{z} = r^2\left(\cos^2\theta + \sin^2\theta\right) = r^2$, as $\cos^2\theta + \sin^2\theta = 1$.

But $|z| = \left|re^{i\theta}\right| = |r|$, as $\left|e^{i\theta}\right| = 1$, so $r^2 = |r|^2 = |z|^2$, as $r \in \mathbb{R}$.

Hence $z\bar{z} = |z|^2$.

It is probably okay to assume that $r \geq 0$.

Question 11 continues on page 6

Prepared by W² Education

(f) $\displaystyle\int_{\frac{3}{2}}^{3} \frac{dx}{\sqrt{6x-x^2}} = \int_{\frac{3}{2}}^{3} \frac{dx}{\sqrt{3^2-(x-3)^2}}$ **3**

$$= \left[\sin^{-1}\left(\frac{x-3}{3}\right)\right]_{\frac{3}{2}}^{3}$$

$$= 0 - \sin^{-1}\left(-\frac{1}{2}\right)$$

$$= \frac{\pi}{6}.$$

(g) (i) As $z = a + ai = a(1+i)$, **1**

$|z| = |a|\sqrt{2}$.

Hence $|z^5| = |z|^5 = 4\sqrt{2}|a|^5 = 4\sqrt{2}a^5$, as $a > 0$.

(ii) As $z^5 = a^5(1+i)^5$ with $a^5 > 0$, $\arg(z^5) = 5\arg(1+i) = \frac{5\pi}{4} + 2k\pi,\ k \in \mathbb{Z}$. **2**

Hence $\text{Arg}(z^5) = -\frac{3\pi}{4} \in (-\pi, \pi]$.

End of Question 11

Please scroll down

Prepared by W² Education

Question 12 (15 marks) Answer Question 12 on pages 8–11 of the Writing Booklet.

(a) (i) Write $z = e^{i\theta}$, so by de Moivre's theorem, $z^n + z^{-n} = 2\cos(n\theta)$, as given. **3**

Then

$$\left(\frac{e^{i\theta} - e^{-i\theta}}{2i}\right)^4 = \left(\frac{1}{2i}\right)^4 \left(z - z^{-1}\right)^4$$

$$= \frac{1}{16}\left(z^4 - 4z^2 + 6 - 4z^{-2} + z^{-4}\right), \text{ as } i^4 = 1$$

$$= \frac{1}{16}\left[\left(z^4 + z^{-4}\right) - 4\left(z^2 + z^{-2}\right) + 6\right]$$

$$= \frac{1}{16}(2\cos 4\theta - 8\cos 2\theta + 6), \text{ with } n = 4 \text{ and } n = 2$$

$$= \frac{1}{8}(\cos 4\theta - 4\cos 2\theta + 3).$$

(ii) As $\left|e^{i\theta}\right| = 1$, $e^{-i\theta} = \overline{e^{i\theta}}$, so $e^{i\theta} - e^{-i\theta} = 2i\,\text{Im}(\cos\theta + i\sin\theta) = 2i\sin\theta$. **2**

Thus, $\dfrac{e^{i\theta} - e^{-i\theta}}{2i} = \sin\theta$.

Hence, by (i),

$$\int \sin^4\theta\, d\theta = \frac{1}{8}\int (\cos 4\theta - 4\cos 2\theta + 3)\, d\theta$$

$$= \frac{\sin 4\theta}{32} - \frac{\sin 2\theta}{4} + \frac{3\theta}{8} + c.$$

Question 12 continues on page 8

W² Education

Prepared by W² Education

(b) Consider $P(z) = (z-2)(z^2-2z+37)(z^2-6z+25)$. **2**

Note that $(z-\alpha)(z-\overline{\alpha}) = z^2 - 2\text{Re}(\alpha)z + |\alpha|^2$, so $(z-(1+6i))$ is a factor of $z^2-2z+37$ and $(z-(3-4i))$ is a factor of $z^2-6z+25$.

Hence 2, $1+6i$ and $3-4i$ are zeroes of P, and P has real coefficients.

Usually HSC spells these as 'zeros', without the 'e'.

(c) As $a, b, c > 0$, $\dfrac{a+7b}{2} \geq \sqrt{7ab} > 0$, $\dfrac{b+2c}{2} \geq \sqrt{2bc} > 0$ and $\dfrac{c+14a}{2} \geq \sqrt{14ca} > 0$. **2**

Multiplying,

$$\frac{1}{8}(a+7b)(b+2c)(c+14a) \geq \sqrt{7ab}\sqrt{2bc}\sqrt{14ca} = \sqrt{14}\sqrt{14}\sqrt{(abc)^2}$$
$$= 14abc, \text{ as } abc > 0.$$

Hence $(a+7b)(b+2c)(c+14a) \geq 8 \times 14abc = 112abc$.

Question 12 continues on page 9

Prepared by W² Education

(d) From the graph, $\ddot{x}(1) = \ddot{x}(3) = 0$, so by symmetry of SHM, the period is $T = 4$. **3**
Thus $\ddot{x}(0) = \pi^2$.

Model the motion by $\ddot{x} = \pi^2 \cos(nt)$, where $\frac{2\pi}{n} = T = 4$, so $n = \frac{\pi}{2}$.

So

$$\frac{d\dot{x}}{dt} = \pi^2 \cos\left(\frac{\pi t}{2}\right)$$

$$\dot{x} = \pi^2\left(\frac{2}{\pi}\right)\sin\left(\frac{\pi t}{2}\right) + k.$$

When $t = 0$, $\dot{x} = 0$, so $k = 0$.

So

$$\dot{x} = \frac{dx}{dt} = 2\pi \sin\left(\frac{\pi t}{2}\right)$$

$$x = -2\pi\left(\frac{2}{\pi}\right)\cos\left(\frac{\pi t}{2}\right) + h.$$

When $t = 0$, $x = -2$, so $h = 2$.

Hence $x(t) = 2 - 4\cos\left(\frac{\pi t}{2}\right)$.

Question 12 continues on page 10

W² Education

Prepared by W² Education

(e) $\displaystyle\int \frac{e^x + e^{2x}}{1 + e^{2x}}\,dx = \int e^x \cdot \frac{1 + e^x}{1 + \left(e^x\right)^2}\,dx$ **3**

$$= \int \frac{d\left(e^x\right)}{1 + \left(e^x\right)^2} + \frac{1}{2}\int \frac{2e^x}{1 + \left(e^x\right)^2}\,d\left(e^x\right)$$

$$= \tan^{-1}\left(e^x\right) + \frac{1}{2}\ln\left(1 + e^{2x}\right) + c, \text{ as } 1 + e^{2x} > 0.$$

End of Question 12

Please scroll down

W2 Education

Question 13 (14 marks) Answer Question 13 on pages 13–16 of the Writing Booklet.

(a) For positive integers n, **2**

$$\begin{aligned}(n+1)^2-(n-1)^2 &= \big((n+1)-(n-1)\big)\big((n+1)+(n-1)\big)\\ &= 2(2n)\\ &= 4n.\end{aligned}$$

Hence any positive multiple of 4, say $4n$, can be written as a difference of squares, namely $(n+1)^2-(n-1)^2$, where both $n+1$ and $n-1$ are integers, as $n \in \mathbb{Z}$.

(b) As $v^2 = -2\big(x^2-4x-4\big)$ **2**

$$\begin{aligned}&= -2(x-2)^2+16\\ &= 0, \text{ at } (x-2)^2 = 8 \text{ or } x = 2 \pm 2\sqrt{2}.\end{aligned}$$

As the particle is in SHM, it oscillates between $x = 2-2\sqrt{2}$ and $x = 2+2\sqrt{2}$ with centre $x = 2$, so the amplitude is $2\sqrt{2}$.

Hence the distance it travels in one period is $4 \times 2\sqrt{2} = 8\sqrt{2}$ m.

Question 13 continues on page 12

Education

Prepared by W² Education

(c) With $\theta = 2\tan^{-1}t$, $d\theta = \dfrac{2dt}{1+t^2}$. **3**

Also, $t = \tan\dfrac{\theta}{2}$, so $\cos\theta = \dfrac{1-t^2}{1+t^2}$ and $\sin\theta = \dfrac{2t}{1+t^2}$.

$$\int \frac{d\theta}{5+3\cos\theta-2\sin\theta} = 2\int \frac{1}{5+\dfrac{3\left(1-t^2\right)}{1+t^2}-\dfrac{2(2t)}{1+t^2}} \times \frac{dt}{1+t^2}$$

$$= 2\int \frac{dt}{5+5t^2+3-3t^2-4t}$$

$$= \int \frac{dt}{t^2-2t+4}$$

$$= \int \frac{d(t-1)}{(t-1)^2+\left(\sqrt{3}\right)^2}$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{t-1}{\sqrt{3}}\right)+c$$

$$= \frac{1}{\sqrt{3}}\tan^{-1}\left(\frac{\tan\dfrac{\theta}{2}-1}{\sqrt{3}}\right)+c.$$

Question 13 continues on page 13

Prepared by W² Education

(d) Let x be the distance fallen from the maximum height, so we seek v at $x = 5\ln(6.76)$. **3**

By Newton's second law,

$$0.8\ddot{x} = 0.8g - 0.08v^2$$

$$\ddot{x} = 10 - \frac{v^2}{10}, \text{ as } g = 10$$

$$v\frac{dv}{dx} = \frac{100 - v^2}{10}$$

$$\int \frac{-2v\,dv}{100 - v^2} = -\frac{1}{5}\int dx$$

$$\ln\left|100 - v^2\right| = -\frac{x}{5} + c$$

$$100 - v^2 = Ae^{-\frac{x}{5}}, \text{ where } A = \pm e^c \in \mathbb{R}.$$

At $x = 0,\ v = 0$ (maximum height), so $A = 100$ and $100 - v^2 = 100e^{-\frac{x}{5}}$.

So $v = 10\sqrt{1 - e^{-\frac{x}{5}}} \geq 0$ (speed).

At $x = 5\ln(6.76)$, the required speed is

$$10\sqrt{1 - e^{-\ln(6.76)}} = 10\sqrt{1 - \frac{1}{6.76}} = 10\sqrt{\frac{5.76}{6.76}}$$

$$= \frac{10\sqrt{576}}{\sqrt{676}} = \frac{10 \times 24}{26}$$

$$= \frac{120}{13} \approx 9.23 \text{ m s}^{-1} \text{ (2 d.p.)}.$$

Question 13 continues on page 14

Prepared by W² Education

(e) (i) Assume for contradiction that $\sqrt{6}$ is rational, so $\sqrt{6} = \frac{a}{b}$, $a, b \in \mathbb{Z}^+$ as $\sqrt{6} > 0$, with a, b coprime. **2**

Then $\frac{a^2}{b^2} = 6$, so $a^2 = 2\left(3b^2\right)$. As $3b^2 \in \mathbb{Z}$, a^2 is even, so a is even (otherwise if a were odd, then a^2 would be odd). ①

Let $a = 2k$, $k \in \mathbb{Z}$, so $4k^2 = 6b^2 \Rightarrow 3b^2 = 2k^2$.

Since 2 and 3 are coprime, 2 divides b^2. That is, b^2 is even. Similarly, by ①, b is also even. ②

From ① and ②, 2 is a common factor of a and b, which contradicts that a, b are coprime.

Hence $\sqrt{6}$ is irrational.

Question 13 continues on page 15

Prepared by W² Education

(e) (ii) Assume for contradiction that $\sqrt{3}-\sqrt{2}$ is rational. **2**

Then $\sqrt{3}-\sqrt{2}=\dfrac{p}{q}$, where $p,\ q\in\mathbb{Z},\ q\neq 0$.

Squaring, $5-2\sqrt{6}=\dfrac{p^2}{q^2}$.

So $\sqrt{6}=\dfrac{5q^2-p^2}{2q^2}$ is rational, as $5q^2-p^2,\ 2q^2\in\mathbb{Z}$ since $p,\ q\in\mathbb{Z}$, and $2q^2\neq 0$ as $q\neq 0$.

This contradicts part (i), as $\sqrt{6}$ is irrational.

Hence $\sqrt{3}-\sqrt{2}$ is irrational.

A near-identical question appeared in our **2023 HSC Premiere**, differing only in part (i), which asked for a proof that $\sqrt{3}$ is irrational.

End of Question 13

Please scroll down

W2 Education

Question 14 (15 marks) Answer Question 14 on pages 18–21 of the Writing Booklet.

(a) Differentiating, $\underset{\sim}{v}(t) = \frac{1}{4}\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. **4**

Consider $d = \left| \underset{\sim}{r}(t) - \begin{pmatrix} 2 \\ 2 \\ 51 \end{pmatrix} \right| = \frac{1}{4}\left| \begin{pmatrix} 12-t \\ 8 \\ t \end{pmatrix} \right|$. The helicopter loses power when $d = 3$.

When $d = 3$,

$$d^2 = \frac{1}{16}\left[(12-t)^2 + 8^2 + t^2\right] = 9$$

$$2t^2 - 24t + 208 = 144$$

$$t^2 - 12t + 32 = 0$$

$$(t-4)(t-8) = 0.$$

The earliest time is $t = 4 < 8$, with $\underset{\sim}{r}(4) = \begin{pmatrix} 4 \\ 4 \\ 52 \end{pmatrix}$.

With no air resistance, resetting $t = 0$ at this point, the helicopter is in projectile motion in the plane $y = 4$. Taking the positive z-axis as upwards, consider the z-component:

$$\ddot{z}(t) = -10$$

$$\dot{z}(t) = -10t + c_1, \text{ but } \dot{z}(0) = \frac{1}{4}, \text{ so } c_1 = \frac{1}{4}$$

$$z(t) = \frac{t}{4} - 5t^2 + c_2, \text{ but } z(0) = 52, \text{ so } c_2 = 52.$$

So at the ground,

$$z(t) = -5t^2 + \frac{t}{4} + 52 = 0$$

$$20t^2 - t - 208 = 0$$

$$(4t-13)(5t+16) = 0.$$

Question 14 continues on page 17

Prepared by W² Education

(a) So $t = \frac{13}{4}$ as $t \geq 0$.

Hence the helicopter takes $\frac{13}{4}$ seconds to reach the ground.

The question omits many essential assumptions. This solution assumes that the ground is the plane $z = 0$, that upwards is the direction of the positive z-axis, and that air resistance can be neglected.

Question 14 continues on page 18

W2 Education

(b) By parts, **3**

$$\int x^2 e^x \, dx = \int x^2 \, d\left(e^x\right) = x^2 e^x - 2\int xe^x \, dx,$$

$$\int xe^x \, dx = \int x \, d\left(e^x\right) = xe^x - \int e^x \, dx = xe^x - e^x.$$

Thus, $\int x^2 e^x \, dx = x^2 e^x - 2xe^x + 2e^x + c.$

As $y^2 = x^2 e^x$,

$$\begin{aligned} V &= \pi \int_0^3 x^2 e^x \, dx \\ &= \pi \left[e^x \left(x^2 - 2x + 2\right) \right]_0^3 \\ &= \pi e^3 (9 - 6 + 2) - \pi(0 - 0 + 2) \\ &= \left(5e^3 - 2\right)\pi \text{ cubic units.} \end{aligned}$$

(c) (i) As $\overrightarrow{BA} = \begin{pmatrix} 4 - 3b \\ 5 - 4b \\ 2 - 4b \end{pmatrix} \perp \ell$ and $\ell \parallel \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix}$, **1**

$$\overrightarrow{BA} \cdot \begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix} = 6(4 - 3b) + 2(5 - 4b) - 3(2 - 4b) = 0.$$

So

$$\begin{aligned} 24 + 10 - 6 &= b(18 + 8 - 12) \\ 14b &= 28. \end{aligned}$$

Hence $b = 2$.

Question 14 continues on page 19

Prepared by W² Education

(c) (ii) As $\overrightarrow{BA} \perp \ell$ and C lies on ℓ, triangle ACB is right-angled at A, so **2**

$$\left|\overrightarrow{AB}\right| = \left|\overrightarrow{BC}\right| \sin \angle ACB$$
$$\left|\overrightarrow{AC}\right| = \left|\overrightarrow{BC}\right| \cos \angle ACB.$$

So $\left|\overrightarrow{AB}\right| = \left|\overrightarrow{AC}\right|$ as $\sin\frac{\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$, since $\angle ACB = \frac{\pi}{4}$.

Now $\overrightarrow{AB} = \begin{pmatrix} 6-4 \\ 8-5 \\ 8-2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$ as $b = 2$ from (i).

So $\left|\overrightarrow{AC}\right| = \left|\overrightarrow{AB}\right| = \sqrt{4+9+36} = 7$.

As $\left|\begin{pmatrix} 6 \\ 2 \\ -3 \end{pmatrix}\right| = 7$, the two possible positions of C are at $\lambda = \pm 1$.

Hence $\overrightarrow{OC} = \begin{pmatrix} 10 \\ 7 \\ -1 \end{pmatrix}$ or $\overrightarrow{OC} = \begin{pmatrix} -2 \\ 3 \\ 5 \end{pmatrix}$.

Question 14 continues on page 20

W2 Education

Prepared by W² Education

(d) (i) Taking the direction of the initial velocity u as positive, the resistance $mk + mv^2$ acts in the negative direction, so by Newton's second law **2**

$$m\ddot{x} = -\left(mk + mv^2\right)$$

$$\ddot{x} = \frac{dv}{dt} = -\left(k + v^2\right)$$

$$\int \frac{dv}{\left(\sqrt{k}\right)^2 + v^2} = -\int dt, \text{ as } k > 0$$

$$\frac{1}{\sqrt{k}} \tan^{-1}\left(\frac{v}{\sqrt{k}}\right) = -t + c.$$

When $t = 0$, $v = u$, so $c = \frac{1}{\sqrt{k}} \tan^{-1}\left(\frac{u}{\sqrt{k}}\right)$.

Hence

$$t = c - \frac{1}{\sqrt{k}} \tan^{-1}\left(\frac{v}{\sqrt{k}}\right)$$

$$= \frac{1}{\sqrt{k}}\left(\tan^{-1}\frac{u}{\sqrt{k}} - \tan^{-1}\frac{v}{\sqrt{k}}\right).$$

Question 14 continues on page 21

W2 Education

Prepared by W² Education

(d) (ii) When $t = \dfrac{\pi}{4\sqrt{k}}$, by (i), **3**

$$\tan^{-1}\frac{u}{\sqrt{k}} - \tan^{-1}\frac{v}{\sqrt{k}} = \frac{\pi}{4}, \text{ as } \sqrt{k} \neq 0 \text{ since } k > 0$$

$$\tan^{-1}\frac{v}{\sqrt{k}} = \tan^{-1}\frac{u}{\sqrt{k}} - \frac{\pi}{4}$$

$$\frac{v}{\sqrt{k}} = \frac{\dfrac{u}{\sqrt{k}} - 1}{1 + \dfrac{u}{\sqrt{k}} \cdot 1}, \text{ as } \tan\frac{\pi}{4} = 1$$

$$v = \sqrt{k}\left(\frac{u - \sqrt{k}}{u + \sqrt{k}}\right).$$

Since $u + \sqrt{k} > 0$, for $v < 0$ we require $u < \sqrt{k}$.

Hence $0 < u < \sqrt{k}$.

But if $u < \sqrt{k}$, the particle comes to rest strictly before $\dfrac{\pi}{4\sqrt{k}}$ seconds, after which its motion ceases. Otherwise, were it to reverse, the force would then act in the same direction as the velocity, thus no longer being a resistive force.

A valid alternative answer is that there are no real values of u for which the particle is moving in the negative direction (but the former is probably intended).

Our **2025 HSC Premiere** has a similar force model in Question 14 (c), set correctly: the constant force was a uniform electric field, which sustains the force even after the droplet stops.

End of Question 14

Please scroll down

Prepared by W² Education

Question 15 (15 marks) Answer Question 15 on pages 23–26 of the Writing Booklet.

(a) For $n = 2$, $\text{LHS} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12} = \frac{14}{24}$ **4**

$\text{RHS} = \frac{13}{24} < \text{LHS}$.

The statement is true for $n = 2$.

Assume true for $n = k$, where k is an integer with $k \geq 2$, so

$$\frac{1}{k+1} + \frac{1}{k+2} + \cdots + \frac{1}{2k} > \frac{13}{24},$$

to prove also true for $n = k + 1$, i.e.,

$$\frac{1}{k+2} + \frac{1}{k+3} + \cdots + \frac{1}{2k+2} > \frac{13}{24}.$$

Consider

$$\begin{aligned}
\frac{1}{k+2} + \frac{1}{k+3} + \cdots + \frac{1}{2k+2} &= \left(\frac{1}{k+1} + \frac{1}{k+2} + \cdots + \frac{1}{2k}\right) + \frac{1}{2k+1} + \frac{1}{2k+2} - \frac{1}{k+1} \\
&> \frac{13}{24} + \frac{1}{2k+1} + \frac{1}{2k+2} - \frac{1}{k+1} \quad \text{using the assumption} \\
&= \frac{13}{24} + \frac{1}{2k+1} - \frac{1}{2k+2} \\
&= \frac{13}{24} + \frac{1}{(2k+1)(2k+2)} \\
&> \frac{13}{24}, \text{ as } (2k+1)(2k+2) > 0 \text{ since } k \geq 2.
\end{aligned}$$

Hence the statement is true by mathematical induction.

A near-identical question was predicted in our **2026 HSC Masterclass** practice paper (HSC-level *Paper 2*): the same sum for all integers $n \geq 1$ with the lower bound $\frac{1}{2}$, for which the base case $n = 1$ gives exactly $\frac{1}{2}$, with essentially the same inductive step.

Question 15 continues on page 23

Prepared by W² Education

(b) By the diagram, as $\angle AMP = 90°$, $\overrightarrow{AM}$ is the projection of $\overrightarrow{AP}$ onto $\overrightarrow{AB}$, where **4**

$$\overrightarrow{AP} = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} \quad \text{and} \quad \overrightarrow{AB} = \begin{pmatrix} 2 \\ 0 \\ 3 \end{pmatrix} - \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

So

$$\overrightarrow{AM} = \frac{3+0+3}{1^2+1^2+1^2}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = 2\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix}.$$

Now $\overrightarrow{AP} + \overrightarrow{PM} = \overrightarrow{AM}$, so

$$\overrightarrow{PM} = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} - \begin{pmatrix} 3 \\ 0 \\ 3 \end{pmatrix} = \begin{pmatrix} -1 \\ 2 \\ -1 \end{pmatrix}.$$

As $\left|\overrightarrow{PM}\right| = \left|\overrightarrow{MQ}\right|$ (M is the midpoint) and P, M, Q are collinear, $\overrightarrow{MQ} = \overrightarrow{PM}$, so

$$\overrightarrow{OQ} = \overrightarrow{OP} + 2\overrightarrow{PM} = \begin{pmatrix} 4 \\ -1 \\ 5 \end{pmatrix} + \begin{pmatrix} -2 \\ 4 \\ -2 \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \\ 3 \end{pmatrix}.$$

Hence Q is the point $(2, 3, 3)$.

Question 15 continues on page 24

W2 Education

Prepared by W² Education

(c) Consider **3**

$$|z_{n+1} - z_n| = \left|\frac{3}{4}iz_n - z_n\right| = |z_n|\left|-1 + \frac{3}{4}i\right| = \frac{|z_n|}{4}|-4 + 3i| = \frac{5}{4}|z_n|.$$

Also $|z_{n+1}| = \left|\frac{3}{4}iz_n\right| = \frac{3}{4}|z_n|$, so $M_k = |z_k|$, $k = 1, 2, 3, \ldots$, is a geometric progression with common ratio $\frac{3}{4}$ and $M_1 = |z_1| = 8$.

Then

$$\begin{aligned}|z_2 - z_1| + |z_3 - z_2| + |z_4 - z_3| + \cdots &= \frac{5}{4}\left(|z_1| + |z_2| + |z_3| + \cdots\right)\\ &= \frac{5}{4}\left(M_1 + M_2 + M_3 + \cdots\right)\\ &= \frac{5}{4}\left(\frac{8}{1 - \frac{3}{4}}\right), \text{ by limiting sum, as } 0 < \frac{3}{4} < 1\\ &= 40.\end{aligned}$$

Hence the sum of the lengths is 40 units.

(d) (i) As $|z_1 - w| = 1$ and $\text{Arg}(z_1 - w) = -\frac{2\pi}{3}$, **2**

$$\begin{aligned}z_1 - w &= e^{-i\frac{2\pi}{3}}\\ &= -\frac{1}{2} - \frac{\sqrt{3}}{2}i.\end{aligned}$$

Hence

$$\begin{aligned}z_1 &= (2 + i) + \left(-\frac{1}{2} - \frac{\sqrt{3}}{2}i\right), \text{ as } w = 2 + i\\ &= \frac{3}{2} + \left(1 - \frac{\sqrt{3}}{2}\right)i.\end{aligned}$$

Question 15 continues on page 25

Prepared by W² Education

(d) (ii) By the diagram, using Pythagoras' theorem, **2**

$$|w|^2 = |W^2| = |z_2|^2 + |z_2 - w|^2, \text{ so } |z_2|^2 = (4+1) - 1 = 2^2$$

and $|z_2| = 2$, as $|z_2| > 0$.

By the diagram, $\operatorname{Arg} z_2 = 2 \operatorname{Arg} w = \operatorname{Arg}(W^2)$.

As the real axis is also a tangent from O, the point representing z_2 is a reflection of $(2, 0)$ in the line through O and w.

Hence

$$\begin{aligned} z_2 &= 2\frac{W^2}{|W^2|} \\ &= \frac{2(2+i)^2}{5}, \text{ as } w = 2 + i \\ &= \frac{2}{5}(3 + 4i) \\ &= \frac{6}{5} + \frac{8}{5}i. \end{aligned}$$

End of Question 15

Please scroll down

Education

Prepared by W² Education

Question 16 (16 marks) Answer Question 16 on pages 28–31 of the Writing Booklet.

(a) As z is on the unit circle, write $z = e^{i\theta}$, where $-\pi < \theta \le \pi$. **4**

Consider $z^3 + z^2 + z = z^2\left(1 + z + z^{-1}\right)$

$= e^{2i\theta}(1 + 2\cos\theta)$, as $z + z^{-1} = e^{i\theta} + e^{-i\theta} = 2\cos\theta$.

So $\text{Im}\left(z^3 + z^2 + z\right) = (1 + 2\cos\theta)\sin 2\theta = 0 \Rightarrow \cos\theta = -\dfrac{1}{2}$ or $\sin 2\theta = 0$.

If $\cos\theta = -\dfrac{1}{2}$, then $\theta = \pm\dfrac{2\pi}{3}$.

If $\sin 2\theta = 0$, then $2\theta = -\pi, 0, \pi, 2\pi$, so $\theta = -\dfrac{\pi}{2}, 0, \dfrac{\pi}{2}, \pi$.

Recalling $z = e^{i\theta}$, the possible values are $z = \pm 1, \pm i, -\dfrac{1}{2} \pm \dfrac{\sqrt{3}}{2}i$.

This factorisation idea, $1 + z + z^2 = z(1 + 2\cos\theta)$ with $|z| = 1$, was the key concept of Question 13 (b) of our **2026 HSC Masterclass** practice paper (HSC-level *Paper 2*), in which z was also placed on the unit circle via $|z| = 1$ and involved relating $\theta = \text{Arg}(z)$ to partial sums of a GP with common ratio z.

(b) (i) Consider $I_n - I_{n-1} = \displaystyle\int_0^1 \left[\frac{1}{\left(x^2+1\right)^n} - \frac{x^2+1}{\left(x^2+1\right)^n}\right] dx$ **1**

$= \displaystyle\int_0^1 \frac{-x^2}{\left(x^2+1\right)^n} dx$, for $n \ge 2$.

Question 16 continues on page 27

Prepared by W² Education

(b) (ii) By definition, **3**

$$I_1 = \int_0^1 \frac{dx}{x^2+1} = \tan^{-1}x\Big|_0^1 = \frac{\pi}{4} - 0 = \frac{\pi}{4}.$$

By (i) with $n = 2$,

$$\begin{aligned} I_2 - I_1 &= \int_0^1 \frac{-x^2}{\left(1+x^2\right)^2}dx \\ &= \frac{1}{2}\int_0^1 x\frac{d}{dx}\left(\frac{1}{1+x^2}\right)dx \\ &= \frac{x}{2\left(1+x^2\right)}\Big|_0^1 - \frac{1}{2}\int_0^1 \frac{1}{1+x^2}dx, \text{ by parts} \\ &= \frac{1}{4} - 0 - \frac{1}{2}I_1. \end{aligned}$$

Hence

$$\begin{aligned} I_2 &= \frac{1}{4} + \frac{1}{2}I_1 \\ &= \frac{1}{4} + \frac{\pi}{8}, \text{ as } I_1 = \frac{\pi}{4}. \end{aligned}$$

This integral $I_n = \int_0^1 \frac{dx}{\left(x^2+1\right)^n}$ was set as Question 14 (a) of our **2026 Trial Premiere**.

Question 16 continues on page 28

Prepared by W² Education

(c) (i) Consider **2**

$$\begin{aligned}
\left|\underset{\sim}{v}(\theta)\right|^2 &= \left(\cos\theta\underset{\sim}{a} + \sin\theta\underset{\sim}{b}\right)\cdot\left(\cos\theta\underset{\sim}{a} + \sin\theta\underset{\sim}{b}\right) \\
&= \cos^2\theta\left(\underset{\sim}{a}\cdot\underset{\sim}{a}\right) + \left(\cos\theta\underset{\sim}{a}\right)\cdot\left(\sin\theta\underset{\sim}{b}\right) + \left(\sin\theta\underset{\sim}{b}\right)\cdot\left(\cos\theta\underset{\sim}{a}\right) + \sin^2\theta\left(\underset{\sim}{b}\cdot\underset{\sim}{b}\right) \\
&= \cos^2\theta\left|\underset{\sim}{a}\right|^2 + 2\cos\theta\sin\theta\left(\underset{\sim}{a}\cdot\underset{\sim}{b}\right) + \sin^2\theta\left|\underset{\sim}{b}\right|^2 \\
&= \cos^2\theta\times 1 + 0 + \sin^2\theta\times 1, \text{ as } \left|\underset{\sim}{a}\right| = \left|\underset{\sim}{b}\right| = 1 \text{ and } \underset{\sim}{a}\perp\underset{\sim}{b} \text{ so } \underset{\sim}{a}\cdot\underset{\sim}{b} = 0 \\
&= \cos^2\theta + \sin^2\theta \\
&= 1.
\end{aligned}$$

Question 16 continues on page 29

Prepared by W² Education

(c) (ii) Let $\underset{\sim}{r}_1(\alpha)$ and $\underset{\sim}{r}_2(\theta)$ be the position vectors of the points on C_1 and C_2. **2**

For the intersection of C_1 and C_2, $\underset{\sim}{r}_1 = \underset{\sim}{r}_2$.

Equating the x and y components,

$$\frac{\cos\alpha}{\sqrt{5}} = \sin\theta,$$
$$\frac{2\cos\alpha}{\sqrt{5}} = \frac{\cos\theta}{\sqrt{2}}.$$

Then $2\sin\theta = \dfrac{\cos\theta}{\sqrt{2}}$, so $\tan\theta = \dfrac{1}{2\sqrt{2}}$ and

$$\sec^2\theta = 1 + \frac{1}{8} = \frac{9}{8} = \frac{1}{\cos^2\theta}.$$

So $\cos\theta = \pm\dfrac{2\sqrt{2}}{3}$, as $\theta \in [0, 2\pi)$.

Now

$$\underset{\sim}{r}_2(\theta) = \frac{\cos\theta}{\sqrt{2}}\begin{pmatrix}0\\1\\1\end{pmatrix} + \sin\theta\begin{pmatrix}1\\0\\0\end{pmatrix}$$
$$= \frac{\cos\theta}{\sqrt{2}}\begin{pmatrix}\sqrt{2}\tan\theta\\1\\1\end{pmatrix}$$
$$= \frac{\cos\theta}{2\sqrt{2}}\begin{pmatrix}1\\2\\2\end{pmatrix}, \text{ where } \frac{\cos\theta}{2\sqrt{2}} = \pm\frac{1}{3}.$$

Hence the points of intersection are $\left(\frac{1}{3}, \frac{2}{3}, \frac{2}{3}\right)$ and $\left(-\frac{1}{3}, -\frac{2}{3}, -\frac{2}{3}\right)$.

The question implies there are at least two intersections.

Question 16 continues on page 30

Prepared by W² Education

(c) (iii) The radius of this circle is $1 = \left|\overrightarrow{OA}\right|$, and it lies in the plane OAP. **4**

Let $\underset{\sim}{a} = \overrightarrow{OA} = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$ and $\underset{\sim}{p} = \overrightarrow{OP} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 3 \\ \sqrt{2} \\ -1 \end{pmatrix}$. The vector $\underset{\sim}{b} = \underset{\sim}{p} - \underset{\sim}{u}$, where $\underset{\sim}{u} = \text{proj}_{\overrightarrow{OA}}\overrightarrow{OP}$, is orthogonal to $\underset{\sim}{a}$ and lies in the plane OAP.

Now

$$\underset{\sim}{u} = \frac{1}{\sqrt{6}}\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 0 \\ \sqrt{2} \\ 0 \end{pmatrix}$$

$$\underset{\sim}{b} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 3 \\ \sqrt{2} \\ -1 \end{pmatrix} - \frac{1}{2\sqrt{3}}\begin{pmatrix} 0 \\ \sqrt{2} \\ 0 \end{pmatrix} = \frac{1}{2\sqrt{3}}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}.$$

As $\underset{\sim}{b}$ is a scalar multiple of $\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$, $\hat{\underset{\sim}{b}} = \frac{1}{\sqrt{10}}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}$.

Hence a vector equation of this circle is

$$\underset{\sim}{v}(\phi) = \frac{\cos\phi}{\sqrt{10}}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + \sin\phi\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}, \quad 0 \le \phi < 2\pi,$$

or

$$\underset{\sim}{v}(\phi) = \cos\phi\begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + \frac{\sin\phi}{\sqrt{10}}\begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix}, \quad 0 \le \phi < 2\pi.$$

End of solutions

Prepared by W² Education

W2 Education

Level 7, 12–14 Ormonde Pde, Hurstville

Burwood Campus OPENING (1 min from station)

ENROLMENTS FOR TERM 4 WELCOME

(Trial a Class)

Tel: (02) 8021 8887

Ph: 0423 712 888

admin@w2education.com.au

www.w2education.com.au

We Win.